A NEW STRATEGY OF SELECTION AND CROSSING IN THE GENETIC ALGORITHMS: APPLICATION TO THE TRAVELLING SALESMAN PROBLEM

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ABSTRACT

To solve TSP, we consider the class of genetic algorithms. One important operation in this algorithm is the selection strategy used to evaluate and select the individuals for crossing. The strategies adopted up to now, are the random selection or through the better circuit length. In this paper, a new model for selection is introduced that operate on the characteristics of the edges of a given circuit (such as the variance of edges or the average). We finally go through some numerical experiments in which we test the efficiency of this new selection strategy.

KEYWORDS

TSP, Genetic Algorithm, Cross-Entropy Method.

1. INTRODUCTION

In the Traveling Salesman Problem(TSP), one has to find a closed tour of minimal length connecting n given cities.TSP is one of the most famous combinatorial optimization problems shown to be NP-hard in the very early development of the complexity theory [10]. The TSP has become a standard tested for combinatorial optimization methods which attempt to find near–optimum solution to this NP-hard problem [7]. Many concrete logistics' applications derive from this problem, which explains why it is so successful.

There are simple algorithm solutions which exist to solve the problem. However, they may become problematic: when dimension increases, the complexity of algorithms increases exponentially and thus, very quickly, computation time is very high. A lot of work in order to find a near perfect solution in a reasonable time has been proposed in the literature, including the classical local search algorithm [1], simulated annealing [11] tabu search[5], elastic nets [3], genetic algorithms [8], [9] and ant colonies [6]. Finally in a recent Ph.D. thesis [1] we find a comparative and experimental study of genetic algorithm applied to TSP.

In this paper, we present a new genetic algorithm approach to the TSP based on a new selection policy for crossing using edges characteristics of individuals instead a random selection.

The paper is organized as follows. Section2 describes the TSP and the genetic algorithm. In section 3 we give our selection strategies for crossing. In section 4 performance results for several TPS instances are presented. Section 5 concludes the paper.

2. TRAVELING SALESMAN PROBLEM AND GENETIC ALGORITHM

Let $G = (V, A, \Omega)$ a graph defined by:

- $V = \{x_0, x_1, ..., x_{n-1}\}$ set of vertex.
- $A = \{x_i x_j\} \in V^2$ set of arcs(edges).
- $\Omega = (\omega_{i,j}) i, j \in [0, n-1]; \omega_{i,j} \in R$ the costs labeling matrix.

A Hamiltonian circuit x_π in G can be defined by:

- $x_\pi = (x_{\pi(0)}, x_{\pi(1)}, ..., x_{\pi(n-1)})$ where π is a permutation on $\{0,1,\ldots,n-1\}$
- The length δ of a circuit x_π can define by $\delta : \rightarrow R;\ \delta(x_\pi) = \left(\sum_{i=0}^{n-2} \omega_{\pi(i),\pi(i+1)} \right) + \omega_{\pi(n-1),\pi(0)}$

The Traveling Salesman Problem consists to find a Hamiltonian cycle x_π which has a minimal length.

Now the Genetic Algorithm (GA) will be given as follows :

```
Algorithm 1
Begin
1.  Compute an initial population g0 of N random individuals(or circuits)
    while stop condition is false do
2.    Select 2 individuals randomly
3.    Construct 2 children with crossover
4.    Add children to the population
5.    Keep the N best individuals in the population
6.    Apply mutation with a probability μ to every individual of the population.
    end while
End. (of algorithm)
```

3. SELECTION STRATEGIES FOR CROSSING

3.1 The entropy Hamiltonian circuit.

We are inspired indirectly by the Cross-Entropy Method [2]. The main question here is the global strategy to apply in order to have the population evolve, this relates to the criteria used to choose the individuals that should be crossod-over (step 2 in the previous algorithm 1). For this we define the entropy function:

Definition : (entropy function) an entropy function $\varphi : \Omega \rightarrow R$ a mathematical function, $\varphi(x_\pi)$ give a real value of R corresponding to the quality (or the entropy) of the circuit x_π.

Now we replace the step 2 by a new one as the selection will be based on best entropy evaluation and the smallest circuit, then we obtain a new scheme of genetic algorithm :

```
Algorithm 2
Begin
1.  Compute an initial population g0 of N random individuals(or circuits)
    while stop condition is false do
2.    Select 2 best individuals: the first with respect to the function of
      entropy the second individual is the smallest circuit;
3.    Construct 2 children with crossover;
4.    Add children to the population;
5.    Keep the N best individuals in the population;
6.    Apply mutation with a probability μ to every individual of the population;
    end while
End.(of algorithm)
```

In the next paragraphs, we shall discuss three strategies of selection for the step 2 in the algorithm 2.

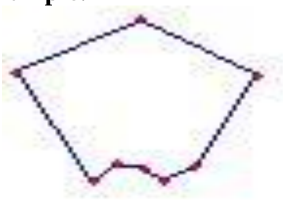
3.2 Entropy by variance strategy

We define the entropy φ of a circuit x_π by the variance function[1] :

$$\varphi(x_\pi) = \frac{1}{n}\left[\left(\sum_{i=0}^{n-2}(\omega_{\pi(i),\pi(i+1)} - \overline{\omega})^2\right) + (\omega_{\pi(n-1),\pi(0)} - \overline{\omega})^2\right] \text{ with } \overline{\omega} = \frac{1}{n}\delta(x_\pi)$$

This method allows us to put in evidence the bad arcs hidden by a simple calculation of the sum of the costs. Nevertheless, it raises a problem as it leads to judge an ingenious individual as being a bad individual if its standard deviation is bad (see figure 1).

Example:



- this individual is optimum
- but it standard deviation (variance) is not the best.

***Figure 1** : Example of individual throwing into question the selection by "entropy"*

3.3 Entropy by partial variances strategy

In order to attenuate the phenomenon of the non selection of a good individual because of its standard deviation (or variance) elevated, we intend to elaborate a similar method that only takes account of a certain part of the arcs in the calculation of the standard deviation. The general idea is to divide the arcs of our individual in categories of length, for example shortest arcs and the longest arcs, then of each of these categories we compute the standard deviation, and finally return an average of these standard deviations.

More formally, if one considers a permutation on {0,1,…,n-1}, then $x_\pi = (x_{\pi(0)}, x_{\pi(1)}, ..., x_{\pi(n-1)})$ become $x_\pi = (x_{\pi(0)}x_{\pi(1)}, x_{\pi(1)}x_{\pi(2)}, ..., x_{\pi(n-1)}x_{\pi(0)})$. We considers now two sets of arcs A_{Min} and A_{Max} , and a real value k such

- $k \in]0,0.5[$
- $A_{Min} \subset x_\pi$ is the set of $E(k \times n)$ shortest arcs of x_π
- $A_{Max} \subset x_\pi$ is the set of $E(k \times n)$ longest arcs of x_π

We define now the functions δ1, δ2, φ1 et φ2 such that :

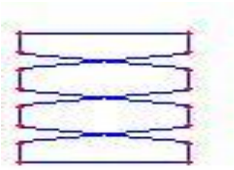
- $\delta_1(x_\pi) = \sum_{x_i x_j \in A_{Min}} \omega_{i,j}$ and $\varphi_1(x_\pi) = \frac{1}{E(k \times n)} \sum_{x_i x_j \in A_{Min}} (\omega_{i,j} - \overline{\omega_1})^2$ with $\overline{\omega} = \frac{1}{E(k \times n)}\delta_1(x_\pi)$
- $\delta_2(x_\pi) = \sum_{x_i x_j \in A_{Max}} \omega_{i,j}$ and $\varphi_2(x_\pi) = \frac{1}{E(k \times n)} \sum_{x_i x_j \in A_{Max}} (\omega_{i,j} - \overline{\omega_1})^2$ with $\overline{\omega} = \frac{1}{E(k \times n)}\delta_2(x_\pi)$

The new entropy function is now : $\varphi(x_\pi) = \frac{\varphi_1(x_\pi) + \varphi_2(x_\pi)}{2}$.

The drawback of this method is that one doesn't know how to distinguish the case where the arcs of a same category of length are close to each other from the case where they are dispersed in our individual. Thus, an

[1] The variance of a random variable is the expectation, or mean, of the squared deviation of that variable from its expected value

individual who systematically alternates short arcs and long arcs will be judged good (see figure 2) whereas it is very bad.
Example:



- The shortest arcs have a weak variance.
- The longest arcs have a weak variance.
- But the individual is not interesting

Figure 2 : *Example of individual throwing into question the selection by partial variance*

3.4 Entropy by partition of population strategy

The two methods that we developed up to now are based on the same principle. Whatever the chosen evaluation, one determines the criteria of quality for a given individual, and then one selects the individuals judged as being of good quality for the crossing. But it raises an essential question: are two individuals both judged as being of good quality necessarily interesting to cross?

Intuitively, one can think that no. Indeed, if one considers an individual *A* as being of good quality even though not optimal, and *B* an individual different from *A* but also of good quality, nothing guarantees that *B* is going to possess the missing gene to *A* to reach the optimality, and vice-versa.

Worse again, to the crossing, the gene of one can damage the interesting part of the gene of the other. Besides, at the time of the development of the method by partial variance, it has first been tested an evaluation on the $E(k \times n)$ longest arcs, then on the $E(k \times n)$ shortest arcs before considering an average of the two. It has been noted then that an evaluation on the variance of the shortest arcs had a tendency to disfavor the variance of the longest arcs. And it is more or less the same for an evaluation of the variance of the longest arcs that tend to disfavor the variance of the shortest arcs.

On the other hand, it seems interesting to cross an individual of weak standard deviation (variance) on its longest arcs with an individual of weak standard deviation (variance) on its shortest arcs. This is the strategy that we intend to adopt (see figure 3).

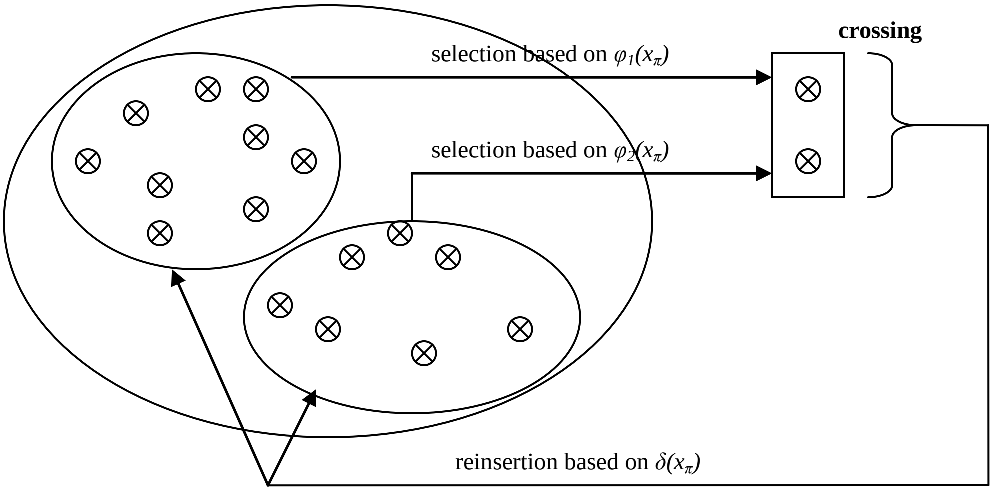


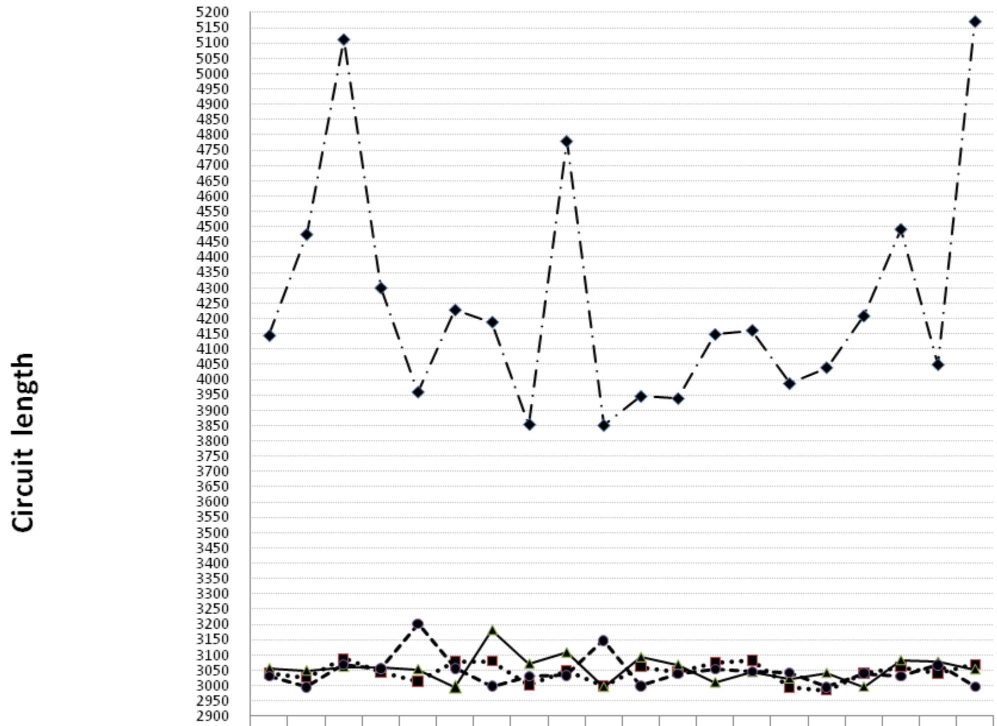
Figure 3 *: Diagram of the global process of evolution on a partition population.*

4. COMPUTATIONAL RESULTS

The algorithms were implemented in Java by a student master project [4]. For all experiments, we set the number of cities to 50. Every case tested will be the subject of 20 successive executions of the algorithm. The crossing strategy used is the method given by Pál K.F 1993 [12]. In all the experiments, the following parameters were used:

- the crossing rate: 60%,
- the mutation rate: 0.2 ,
- the selection is by roulette method,
- the population size is 100 individuals,

The results shown in table 1 give the 20 successive TPS tests (1, 2, …20), in each test we find the application of four selection strategies (random strategy, entropy by variance strategy, entropy by partial variances strategy, and entropy by partition of population strategy).



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Random selection strategy	4144	4475	5113	4300	3959	4228	4188	3853	4780	3850	3945	3938	4148	4161	3987	4039	4208	4492	4049	5172
Entropie by variance strategy	3040	3026	3088	3043	3014	3078	3080	3002	3048	2999	3060	3045	3076	3082	2995	2985	3040	3060	3040	3070
Entropie by variances partiels strategy	3056	3049	3061	3058	3053	2996	3182	3071	3108	2995	3093	3068	3010	3044	3023	3041	2996	3083	3078	3053
Entropie by Partition of population strategy	3030	2995	3070	3055	3201	3056	2998	3031	3033	3147	2999	3039	3054	3046	3043	2995	3040	3032	3067	2996

Table 1: *The 20 successive executions of the algorithm*

The table 2 is a synthesis of the table 1 in which the following indicators are used:

- average: we simply calculate an average of the 20 results returned. This indicator informs us directly on the quality of the solution returned,
- variance: we calculate here the variance of the 20 results, more this one will be low, more it will mean that the algorithm will be good in terms of reproducibility of the solution found,
- gap to the best known result: based on the average of the 20 tests and on the best solution known, we determine the" middle distance" remaining to reach the best solution known,
- we give also the better circuit and the bad circuit.

strategy of selection	the better circuit	the bad circuit	average	variance	gap to the best optimum known
random selection strategy	3850	5172	4251.45	372.14	42.43%
entropy by variance strategy	2985	3088	3043.55	30.82	1.96%
entropy by variances partials strategy	2995	3182	3055.90	42.50	2.38%
entropy by Partition of population strategy	2995	3201	3046.35	49.32	2.06%

Table 2: *The Results on the evaluation strategies*

The numerical tests are extremely conclusive. Nevertheless, the algorithm remains perfectible enough since one is on average to 1.96% near the best optimum known (see table 2, line 2: entropy by variance strategy). Also recall that at least 23 different solution to less of 2% of the best solution known.

5. CONCLUSION

We presented a genetic algorithm as a solution of the TSP, using a novel selection mechanism to select the individuals that should be crossed-over. The numerical experiments clearly indicate advantages and limitations of our algorithm.

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